GCSE Maths – Algebra

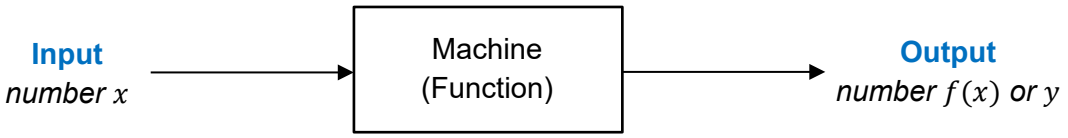
Functions

Notes

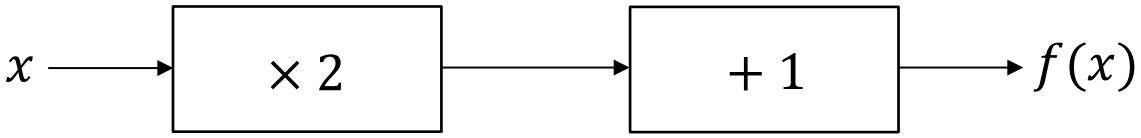
WORKSHEET

Functions

A **function** can be thought of as a machine that takes in a number, performs operations on it and outputs a different number.



Functions are written in the form $f(x)$ which means 'function with an input number x'. A function $f(x)$ is typically written in algebra, for example $f(x) = 2x + 1$. We must be careful to apply the order of operations correctly. To do this, we use **BIDMAS** (Brackets, Indices, Division, Multiplication, Addition, Subtraction).



We could be asked to **replace** x with a specific number to evaluate a function at a certain value.

Example: What is the value of $f(x) = 2x + 1$ when $x = 2$?

1. We **replace** x with the number 2.

$f(x)$ evaluated at $x = 2$ is equal to $f(2)$:

$$f(2) = 2(2) + 1$$

2. **Simplify** this where possible and compute the required computation.

$$f(2) = 2(2) + 1 = 4 + 1 = 5$$

This type of question can be asked in many different ways. For example, we may simply be asked to find $f(2)$. This similarly means we must **replace** x with the number 2.

Inverse Functions (Higher Only)

Lots of **functions** have **inverse functions**. These do the opposite of what a **function** does to a number x. They are written in the form $f^{-1}(x)$.

A common exam question is to find the **inverse function** of a **function**. The following example demonstrates the method required to do that.

Example: What is the inverse function of $f(x) = 3x - 7$?

1. Replace $f(x)$ with y.

$$y = 3x - 7$$

2. Rearrange the equation to make x the **subject**.

$$y = 3x - 7$$

$$y + 7 = 3x$$

$$x = \frac{y + 7}{3}$$

3. Swap the letters x and y with each other and then replace y with $f^{-1}(x)$.

$$y = \frac{x + 7}{3} \quad \rightarrow \quad \boldsymbol{f^{-1}(x) = \frac{x + 7}{3}}$$

We can think of the **inverse function** as the **reverse** process.

Composite Functions (Higher Only)

When we have more than one **function**, we can combine them to form a **composite function**. Composite functions formed from functions $f(x)$ and $g(x)$ may take the form $fg(x)$ or $gf(x)$, depending on which function is substituted into the other.

Example: If $f(x) = 2x + 1$ and $g(x) = 6x^2 - 4$, what is $fg(x)$?

1. **Substitute** the innermost function, $g(x)$, into the outer function, $f(x)$, in the place of the letter x.

Putting $g(x)$ into $f(x)$ gives

$$fg(x) = f\big(g(x)\big) = 2\big(g(x)\big) + 1 = 2(6x^2 - 4) + 1$$

2. **Expand** any brackets being multiplied and **simplify** the equation where possible.

$$fg(x) = 12x^2 - 8 + 1 = 12x^2 - 7$$

$$\boldsymbol{fg(x) = 12x^2 - 7}$$

We may even be asked to find the value of a **composite function** when x has a specific value. To do this, it is helpful to first write the **composite function** algebraically before then **substituting** in a value of x.

Example: If $f(x) = x^2 - 5$ and $g(x) = 6x - 4$, what is $fg(3)$?

1. **Substitute** the innermost function in the place of x into the outer function.

$$fg(x) = f\big(g(x)\big) = \big(g(x)\big)^2 - 5 = (6x - 4)^2 - 5$$

2. **Expand** any brackets being multiplied and **simplify** where possible.

$$(6x - 4)^2 - 5 = (36x^2 - 48x + 16) - 5$$

$$\boldsymbol{fg(x) = 36x^2 - 48x + 11}$$

3. Substitute the given x-value, $x = 3$, into this equation and calculate the required value.

$$fg(3) = 36(3)^2 - 48(3) + 11 = 324 - 144 + 11 = \mathbf{191}$$

Functions – Practice Questions

1. Complete the following:

   a) Find the value of $f(5)$ when $f(x) = 3x - 2$.

   b) Find the value of $g(7)$ when $g(x) = 4x - x^2$.

2. Solve the following for x:

   a) $h(x) = x^2 - 5x + 2,\ \ h(x) = -4$.

   b) $f(x) = 18x - 2,\ f(x) = 6$.

3. Find the inverse functions of the following functions:

   a) $f(x) = 4x + 7$

   b) $g(x) = 15x^2 + 3$

   c) $f(x) = \frac{2x}{3+x}$

4. Find the value of $g(x) = \frac{9-9x^2}{x^2}$ if $g^{-1}(7)$.

5. Solve $h(x) = 3x^2 + 6$ for x given $h^{-1}(x) = 2$.

6. Given $f(x) = 1 - 2x^3$ and $g(x) = \frac{3}{x} - 4$ show that $gf(x) = \frac{8x^3-1}{1-2x^3}$.

7. Given $f(x) = 4x + 6$ and $g(x) = x^2 - 9$ find the value of $fg(3)$.

Worked solutions for the practice questions can be found amongst the worked solutions for the corresponding worksheet file.